ENEL 697 Digital Image Processing

Test No. 1. Winter 2004

1. Spatial resolution may be expressed in terms of any of the following
   - sampling interval (in mm, cm, etc.)
   - the smallest visible object or separation between objects (mm, μm)
   - the finest grid pattern that remains visible (line pairs / mm)

(see p 99, Section 2.10 of textbook)

Gray-scale resolution depends upon the number of quantization levels available and how they are mapped to the dynamic range of the input data. For example, if 8 bits / pixel are used, we have 256 levels available. If this range is mapped to cover optical density (OD) in the range 0.1 – 2.9, each gray level represents $(2.9 - 0.1) / 256 = 0.11$ OD.

(see pp 66 – 69, section 2.3.2)

2. $FT[f_1(x,y)]$

$$= \int\int_{-\infty}^{\infty} f(x-x_1, y-y_1) \exp[-j2\pi(ux+vy)]\,dx\,dy$$

Let $x - x_1 = \alpha$, $y - y_1 = \beta$.
Then $x = \alpha + x_1$, $y = \beta + y_1$, $dx = d\alpha$, $dy = d\beta$.
The limits remain $\pm\infty$.

$FT[f_1(x,y)]$

$$= \int\int_{-\infty}^{\infty} f(\alpha, \beta) \exp[-j2\pi(u\alpha + ux_1 + v\beta + vy_1)]\,d\alpha\,d\beta$$

$$= \exp[-j2\pi(ux_1 + vy_1)] \times$$

$$\int\int_{-\infty}^{\infty} f(\alpha,\beta) \exp[-j2\pi(u\alpha + v\beta)]\,d\alpha\,d\beta$$

$$= \exp[-j2\pi(ux_1 + vy_1)]\; F(u,v)$$

where $F(u,v) = FT[f(x,y)]$.

In the space domain, $f_1(x,y)$ is given by shifting $f(x,y)$ by (x_1, y_1). This causes an additional linear phase component in the FT (spectrum) as indicated by the term $\exp[-j2\pi(ux_1 + vy_1)]$.
The magnitude of the spectrum is not affected by the shift or translation; that is $|F_1(u,v)| = |F(u,v)|$.

$$\angle F_1(u,v) = \angle F(u,v) - 2\pi(ux_1 + vy_1).$$

Note: (x_1, y_1) are constants.

3. Remember that, before doing the 'mask' operation, the impulse response image should be flipped or reversed in x and/or y!

(a) $[-1 \quad 1]$ reversed to $[1 \quad \overset{*}{-1}]$

Result: $\begin{bmatrix} -1 & -1 & 1 & 1 \\ -2 & -1 & 1 & 2 \\ -1 & -1 & 1 & 1 \end{bmatrix}$

$*$: origin or reference point

may be omitted because the problem asked for only a 3×3 output.

(b) $[-1 \quad 1]^T = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ reversed to $\begin{bmatrix} 1 \\ -1^* \end{bmatrix}$

Result: $\begin{bmatrix} -1 & -2 & -1 \\ -1 & -1 & -1 \\ 1 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$

may be omitted

(c) reversal has no effect on $h(m,n)$.

Result:

$$\frac{1}{9} \times \begin{bmatrix} 1 & 3 & 4 & 3 & 1 \\ 3 & 8 & 11 & 8 & 3 \\ 4 & 11 & 15 & 11 & 4 \\ 3 & 8 & 11 & 8 & 3 \\ 1 & 3 & 4 & 3 & 1 \end{bmatrix}$$

adequate to give this 3×3 part.

4. Laplacian operator output

$$g(x,y) = f(x,y-1) + f(x-1,y) + f(x,y+1) + f(x+1,y) - 4f(x,y)$$

From Problem 1, we have

$FT[f(x,y-1)] = \exp(-j2\pi v) F(u,v)$

$FT[f(x,y+1)] = \exp(+j2\pi v) F(u,v)$

$FT[f(x-1,y)] = \exp(-j2\pi u) F(u,v)$

$FT[f(x+1,y)] = \exp(+j2\pi u) F(u,v)$

Transfer function or MTF

$$H(u,v) = G(u,v) \div F(u,v)$$
$$= \exp(-j2\pi v) + \exp(+j2\pi v) + \exp(-j2\pi u) + \exp(+j2\pi u) - 4$$

Now, $\exp(-j2\pi v) + \exp(+j2\pi v) = 2\cos(2\pi v)$

$\exp(-j2\pi u) + \exp(+j2\pi u) = 2\cos(2\pi u)$

$\therefore H(u,v) = 2[\cos(2\pi u) + \cos(2\pi v) - 2]$

for $(u,v) = (0,0)$ to $(1,1)$

or $(-0.5,-0.5)$ to $(0.5,0.5)$.

in normalized frequency.

At $(u,v) = (0,0)$, $H(u,v) = 0$: DC removed.

At $(u,v) = (0.5,0.5)$, $H(u,v) = -8$: maximum gain at the highest frequency.

This is a highpass filter.

4. continued.

a) In the space domain, the filter results in zero output in areas of constant value (brightness) and high output across edges. Noisy pixels will also result in high output.
The Laplacian is the second derivative (difference operator). The output will have +ve and -ve values, with the average = 0. Edges will be highlighted in the output.

b) In the frequency domain, the Laplacian has a highpass filter characteristic. High-frequency components get boosted - this enhances edges but also increases noise. The (0,0) frequency or DC response is zero.

5. Mean: $\frac{1}{9}(52+59+41+62+74+66+56+57+59) = \underline{\underline{58.44}}$

median : sort the pixels in ascending order.

41, 52, 56, 57, 59, 59, 62, 66, 74

pick the 5^{th} value (in the middle of the array).

output = $\underline{\underline{59}}$.