UNIVERSITY OF CALGARY
DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING
ENEL697 DIGITAL IMAGE PROCESSING
TEST NO. 1
WINTER 2005 SESSION
7 March 2005

**Instructions:**

1. This is a closed-book, closed-notes test.
2. The use of only a nonprogrammable calculator with no text storage facilities is permitted.
3. Answer all five questions.
4. Total marks = 20.
5. Time permitted = 90 minutes.

**Question 1:** Give the typical units for the variables $(x, y)$ and $(u, v)$ used in the representation of images in the space and frequency domains.
(2 marks)

**Question 2:** Draw schematic sketches of the histograms of the following types of images:
(a) A collection of objects of the same uniform gray level placed on a uniform background of a different gray level.
(b) A collection of relatively dark cells against a relatively bright background, with both having some intrinsic variability of gray levels.
(c) An under-exposed X-ray image.
(d) An over-exposed X-ray image.
Annotate the histograms with labels and comments.
(4 marks)

**Question 3:** Define two rectangular functions as

$$\begin{aligned} f_1(x,y) &= 1 \quad \text{if } 0 \le x \le X; 0 \le y \le Y \\ &= 0 \quad \text{otherwise,} \end{aligned} \tag{1}$$

and

$$f_2(x,y) = \begin{cases} 1 & \text{if } |x| \le \frac{X}{2},\ |y| \le \frac{Y}{2} \\ \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Starting from the definition of the 2D Fourier transform, derive the Fourier transforms $F_1(u, v)$ and $F_2(u, v)$ of the two functions; show all steps.
Explain the differences between the two functions in the spatial and frequency domains.
(6 marks)

**Question 4:** Using the continuous 2D convolution and Fourier transform expressions, prove that convolution in the space domain is equivalent to multiplication of the corresponding functions in the Fourier domain.
(4 marks)

**Question 5:**
The $5 \times 5$ image

$$f(m,n) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 10 & 10 & 0 \\ 0 & 10 & 10 & 10 & 0 \\ 0 & 10 & 10 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

is processed by two systems in cascade. The first system produces the output $g_1(m,n) = f(m,n) - f(m-1,n)$. The second system produces the output $g_2(m,n) = g_1(m,n) - g_1(m,n-1)$.

Compute the images $g_1$ and $g_2$.

Does the sequence of application of the two operators affect the result? Why (not)?

Explain the effects of the two operators.
(4 marks)

******